Progressive Mathematics Initiative® (PMI®)
MATH6436: Learning and Teaching Trigonometry

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Course Credit: 3.0 NJCTL credits

Dates & Times:
This is a 3-credit, self-paced course, covering 9 modules of content. The exact number of hours that you can expect to spend on each module will vary based upon the module coursework, as well as your study style and preferences. You should plan to spend approximately 15 hours per credit working online, and up to 30 hours per credit working offline.

Graduate Student Handbook: www.njctl.org/graduate-handbook/

COURSE DESCRIPTION:

This course is for teachers to learn the content of PMI Trigonometry and how to teach that course to students. It provides graphing & analysis techniques to develop an understanding of more complex relations, functions, and coordinates that require trigonometry. The verification of trigonometric identities and inverse relations through algebraic reasoning are included. Topics include an overview of trigonometric functions; analytic trigonometry; conic sections; polar coordinates & parametric equations; vectors; exponential & logarithmic functions; and sequences & series.

STUDENT LEARNING OUTCOMES:

Upon completion of the course, the student will be able to:

1. Demonstrate an understanding of mathematics concepts of Trigonometry, including how to teach them, detailed in the module learning outcomes below.

2. Integrate PMI materials (including presentations, labs, practice problems, etc.) to support student learning and deliver effective instruction.
3. Create a social constructivist learning environment through the use of formative assessment questions, interpreting the results of this assessment to effectively facilitate student-led discussions that support deeper understanding of the content.
4. Integrate multiple attempts to demonstrate student mastery of content knowledge, as encouraged/fostered by the PMI pedagogy.
5. Implement learning plans that are aligned to Common Core and Pre-AP standards and allow for differentiation based on the needs of learners.

TEXTS, READINGS, INSTRUCTIONAL RESOURCES:
Required Texts:

- This course uses a free digital textbook accessible at: https://moodle.njctl.org/course/view.php?id=207
- Participants will download SMART Notebook presentations, homework files, labs, and teacher resources.

Recommended Readings:

- Related articles within discussion prompts

COURSE REQUIREMENTS:
In order to receive a Passing grade, the participant must complete the following course requirements:

1. Activities: A number of different learning activities will ensure participant engagement and learning in the course. These include:
   - Engage in video module lessons which demonstrate minimized direct instruction followed by frequent formative assessment.
   - Completion of formative assessments aligned to learning objectives which include detailed analysis when answered incorrectly.
   - Interaction with module discussion boards that allow conversation with peers and course instructors about the module's content, delivering that content to students. Discussion boards also serve as a place to ask and answer questions related to the module's content.
2. Short Answer Assignment: Each module requires one (1) original response to a given prompt. These prompts are typically based upon course lessons and require teachers to analyze, reflect, and make connections between the module's content and their own classroom practice.
3. Mastery Exercises: For each module, these multiple-choice question quizzes assess the content knowledge gained in a module. Participants have the opportunity to retake; random questions are pulled from a larger question bank on each attempt ensuring varied questions.
4. Virtual Labs: In each module, a virtual lab write-up will be submitted. Virtual labs are interactive lab simulations that promote discovery-based student learning through real-world applications and analysis.

5. Module Exam: One is completed at the end of each module. It is a culminating exam consisting of multiple choice and free response questions aligned to the standards and objectives of the module.
6. Reflection Paper: At the end of the course, participants are required to reflect on the knowledge taught in the course, make connections, and compare/contrast their current pedagogy with new strategies gained in this assignment.
7. Final Exam: At the end of the course, a comprehensive exam consisting of Multiple Choice and Free Response questions assesses the content knowledge learned throughout the course.

GRADE DISTRIBUTION AND SCALE:

Grade Distribution:

Module Exams	70%
Final Exam	10%
Labs	6%
Short Answer Assignments	6%
Mastery Exercises	6%
Reflection Paper	2%

Grade Scale:

A	93 – 100
A-	90 – 92
B+	86 – 89
B	83 – 86
B-	80 – 82
C+	77 – 79
C	73 – 76
C-	70 – 72
D	60.0 – 69.9
F	59.9 or below

GRADING RUBRIC:

The following rubric is used to score:

- Short Answer Assignment – 6% of grade
- Reflection Paper – 2% of grade

The minimum possible score for this rubric is 4 points, and the score will be converted to the minimum grade available in this module (which is zero unless the scale is used). The maximum score of 25 points will be converted to the maximum grade.

Intermediate scores will be converted respectively and rounded to the nearest available grade. If a scale is used instead of a grade, the score will be converted to the scale elements as if they were consecutive integers.

	Meets Expectation	**Approaches Expectation**	**Below Expectation**	**Limited Evidence**
	7 points	***5 points***	***3 points***	***1 point***
Content	• Demonstrates excellent knowledge of concepts, skills, and theories relevant to topic.	• Demonstrates fair knowledge of concepts, skills, and theories.	• Demonstrates incomplete or insubstantial knowledge of concepts, skills, and theories.	• Demonstrates little or no knowledge of concepts, skills, and theories.
Depth of Reflection	• Content is well supported and addresses all required components of the assignment.	• Content is partially supported; addresses most of the required components of the assignment.	• Content contains major deficiencies; addresses some of the required components of the assignment.	• Content is not supported and/or includes few of the required components of the assignment.
Evidence and Practice	• Response shows strong evidence of synthesis of ideas presented and insights gained throughout the entire course. The implications of these insights for the respondent's overall teaching practice are thoroughly detailed, as applicable.	• Writing is mostly clear, concise, and well organized with good sentence/paragraph construction. Thoughts are expressed in a coherent and logical manner. There are no more than five spelling, grammar, or syntax errors per page of writing.	• Response is missing some components and/or does not fully meet the requirements indicated in the instructions. Some questions or parts of the assignment are not addressed. Some attachments and additional documents, if required, are missing or unsuitable for the purpose of the assignment.	• Response excludes essential components and/or does not address the requirements indicated in the instructions. Many parts of the assignment are addressed minimally, inadequately, and/or not at all.
	4 points	***3 points***	***2 points***	***1 point***
Writing Quality	Writing is well-organized, clear, concise, and focused; no errors.	Some minor errors or omissions in writing organization, focus, and clarity.	Some significant errors or omissions in writing organization, focus, and clarity.	Numerous errors in writing organization, focus, and/or clarity.

The following rubric is used to score:
- Labs – 6% of grade

The minimum possible score for this rubric is 2 points, and the score will be converted to the minimum grade available in this module (which is zero unless the scale is used). The maximum score of 14 points will be converted to the maximum grade.

Intermediate scores will be converted respectively and rounded to the nearest available grade. If a scale is used instead of a grade, the score will be converted to the scale elements as if they were consecutive integers.

	Meets Expectation	**Approaches Expectation**	**Below Expectation**	**Limited Evidence**
	7 points	***5 points***	***3 points***	***1 point***
Completeness	• Lab write-up is complete with no missing fields.	• Lab write-up has 1-2 missing fields.	• Lab write-up has 3-5 missing fields.	• There are more than 5 missing fields on the lab write-up.
Calculations	• All answers are calculated correctly.	• Most answers are calculated correctly, but there are 1-2 minor calculation errors.	• Most answers are calculated correctly, but there are multiple minor calculation errors, or 1-2 gross miscalculations.	• There are calculation errors throughout the lab.

The remaining types of assignments are not scored using a rubric. These assignments are scored using percentage correct to assign a letter grade. The assignments in this manner are as follows:
- Mastery Exercises – 6% of grade
- Module Exams – 70% of grade
- Final Exam – 10% of grade

Mastery Exercises can be retaken as many times as desired to ensure a high score. Due to the nature of these assignments, each time they are taken, they will be composed of unique questions pulled randomly from a larger question bank.

Module and Final Exams are scored using a curve, which allows us to keep content exams rigorous. Module Exams can be retaken one time. Final Exams cannot be retaken.

ACADEMIC STANDING:
NJCTL has established standards for academic good standing within a student's academic program. Students enrolled in any NJCTL online course must receive an 80 or higher to successfully complete a course and receive credit for that course. An 80 is equivalent to a GPA

of 2.7 or B-. Additionally, students in an endorsement program must receive a cumulative GPA of 3.0 for all courses combined in order to successfully complete the program.

ACADEMIC INTEGRITY:
Students must assume responsibility for maintaining honesty in all work submitted for credit and in any other work designated by the instructor of the course. Academic dishonesty includes cheating, fabrication, facilitating academic dishonesty, plagiarism, reusing /repurposing your own work, unauthorized possession of academic materials, and unauthorized collaboration.

CITING SOURCES WITH APA STYLE:
All students are expected to follow proper writing and APA requirements when citing in APA (based on the APA Style Manual, 6th edition) for all assignments.

DISABILITY SERVICES STATEMENT:
We are committed to providing reasonable accommodations for all persons with disabilities. Any student with a documented disability requesting academic accommodations should contact the Dean of Students, Melissa Axelsson, for additional information to coordinate reasonable accommodations for students with documented disabilities (melissa@njctl.org).

NETIQUETTE:
Respect the diversity of opinions among the instructor and classmates and engage with them in a courteous, respectful, and professional manner. All posts and classroom communication must be conducted in accordance with the student code of conduct. Think before you push the Send button. Did you say just what you meant? How will the person on the other end read the words?

Maintain an environment free of harassment, stalking, threats, abuse, insults or humiliation toward the instructor and classmates. This includes, but is not limited to, demeaning written or oral comments of an ethnic, religious, age, disability, sexist (or sexual orientation), or racist nature; and the unwanted sexual advances or intimidations by email, or on discussion boards and other postings within or connected to the online classroom.

If you have concerns about something that has been said, please let your instructor know.

CLASS SCHEDULE:

Module	Module Learning Outcomes	Assignments
1 – Trigonometry	• Solve right triangles using trigonometric ratios and inverse trigonometric ratios. • Convert radians to degrees and degrees to radians. • Use the unit circle to find the value of trigonometric functions at any angle. • Derive the graphs of the sine and cosine functions using their values on the unit circle. • Compare the graphs and/or equations of 2 trigonometric functions to determine any transformations that map the original function to the new function. • Model and solve real-world problems using the trigonometric functions.	• Short Answer Assignments • Lab • Mastery Exercises • Module Exam
2 – Trigonometric Functions	• Evaluate the circumference of a circle given the diameter/area. • Evaluate the arc length of a circle given the diameter. • Convert between radians and degrees. • Evaluate sin/cos/tan given a right triangle. • Evaluate sin/cos/tan given the value of another one of the trigonometric ratios. • Evaluate missing side lengths of special right triangles. • Evaluate trigonometric functions for right triangles that are created by a point on a coordinate plane. • Identify what quadrants produce positive/negative trigonometric functions in the unit circle. • Utilize the unit circle to find the value of trigonometric functions of any angle. • Solve for missing values in sine/cosine functions. • Arrange sine/cosine values in numeric order. • Graph sine/cosine functions. • Describe transformations of the graphs for sine/cosine functions. • Create formulas/graphs for transformed sine/cosine functions. • Graph the tangent function. • Describe the relationship between sine/cosine/tangent and their reciprocals. • Graph secant/cosecant/tangent functions. • Create formulas/graphs for transformed secant/cosecant/cotangent functions. • Describe the relationship between sine/cosine/tangent and their inverse, arcsine/arccosine/arctangent. • Evaluate period, frequency, and amplitude for simple harmonic motions.	• Short Answer Assignments • Lab • Mastery Exercises • Module Exam

3 – Analytic Trigonometry	• Transform trigonometric functions to create trigonometric identities. • Simplify trigonometric functions using Pythagorean identities and negative angle identities. • Utilize Pythagorean identities and negative angle identities to complete proofs. • Evaluate the exact value of given trigonometric expressions using sum/difference identities. • Simplify trigonometric functions using sum/difference identities, cofunction identities and supplement angle identities. • Utilize sum/difference identities, cofunction identities and supplement angle identities to complete proofs. • Simplify trigonometric functions using double/half angle identities. • Transform trigonometric functions using power reducing identities. • Transform trigonometric functions using product-sum identities. • Solve trigonometric equations using trigonometric identities. • Evaluate missing sides/angles of non-right triangles using the laws of Sine/Cosine. • Evaluate the number of triangles that meet a certain set of criteria using the laws of Sines/Cosines.	• Short Answer Assignments • Lab • Mastery Exercises • Module Exam

4 – Conic Sections	● Evaluate the midpoint of a line segment and the distance between two given points. ● Utilize midpoint/distance formulas to find endpoints. ● Identify parabola graphs from given equations. ● Identify key components of a parabola from its equation/graph; vertex, direction of opening, etc. ● Convert the equation of a parabola into different forms; standard/general form, intercept form, and vertex form. ● Identify the eccentricity of conic sections. ● Identify the transformations of the parent parabola function to create a given parabola's graph/equation. ● Evaluate the focal distance, focus, directrix, and the axis of symmetry of a parabola's graph/equation. ● Graph parabolas. ● Identify the center and length of the radius from the given equation of a circle. ● Create the equation of a circle from a given graph. ● Create the equation of a circle given two or more components; center, points on the circle, diameter endpoints, etc. ● Convert the equation of a circle into different forms; standard form, general form, etc. ● Identify key components of an ellipse from its graph/equation; major/minor axes, vertices, co-vertices, etc. ● Evaluate the foci and the length of the major/minor axes of an ellipse. ● Graph an ellipse. ● Create the equation of an ellipse given two or more components; foci, axis length, center, etc. ● Convert the equation of an ellipse into different forms; standard form, general form, etc. ● Identify key components of a hyperbola from its graph/equation; vertices, foci, asymptotes, etc. ● Graph a hyperbola given in standard form. ● Convert the equation of a hyperbola into different forms; standard form, general form, etc. ● Identify conic sections and their key components from an equation given in general form.	● Short Answer Assignments ● Lab ● Mastery Exercises ● Module Exam

5 – Polar Coordinates & Parametric Equations	● Identify and graph points on a polar plane. ● Convert coordinates between polar and rectangular form. ● Identify polar graphs from their equations. ● Graph equations on the polar plane. ● Identify and prove symmetry of a given equation's graph. ● Identify key features of a rose curve graph; petals, length of petal, etc. ● Graph lemniscates on the polar plane. ● Graph parametric equations on a rectangular coordinate plane. ● Convert parametric equations between polar and rectangular form. ● Eliminate the parameter to graph parametric equations on the rectangular coordinate plane. ● Create parametric equations to match a given set of conditions or graphs. ● Convert parametric equations from rectangular to polar form. ● Create parametric equations to model projectile, linear and circular motion.	● Short Answer Assignments ● Lab ● Mastery Exercises ● Module Exam
6 – Vectors	● Identify, name, and draw vectors. ● Evaluate the horizontal and vertical components of a vector drawn on a coordinate plane. ● Calculate the magnitude and direction of a vector. ● Create vectors from verbal descriptions and evaluate their magnitude and direction. ● Evaluate scalar multiples of vectors. ● Evaluate the resultant, magnitude, and direction of two vectors that form a right angle. ● Evaluate the resultant, magnitude, and direction of two vectors that do not form a right angle. ● Create the negative of a vector. ● Use the negative of a vector to subtract vectors. ● Write vectors in component form. ● Add/subtract vectors in component form. ● Create the equation of a vector in standard position. ● Create the vector/parametric equation of a line that is parallel to a vector in the standard position and that passes through two points. ● Create the parametric equation of a line when given the vector equation. ● Evaluate the dot product of given vectors. ● Determine if two vectors are orthogonal. ● Determine if two vectors form an obtuse or acute angle based on their dot product. ● Apply properties/operations of vectors to solve real-life application problems.	● Short Answer Assignments ● Lab ● Mastery Exercises ● Module Exam

7 – Exponential & Logarithmic Functions	• Simplify exponential expressions when they have a common base. • Rewrite exponential expressions that have different bases as exponential expressions that have a common base. • Graph and identify characteristics of graphs of exponential functions. • Solve real-world problems involving exponential growth and decay. • Convert algebraic expressions from exponential form to logarithmic form and vice-versa. • Solve exponential and logarithmic equations. • Use properties of logarithms to solve equations. • Solve real-world problems using the rules of exponents and properties of logarithms.	• Short Answer Assignments • Lab • Mastery Exercises • Module Exam
8 – Sequences & Series	• Identify sequences of numbers and determine whether a sequence is finite or infinite. • Identify the common difference in an arithmetic sequence. • Identify the common ratio in a geometric sequence. • Find the value missing term(s) in arithmetic and geometric sequences. • Write arithmetic and geometric sequences using both the recursive and explicit formulae. • Identify special sequences, including Fibonacci Sequences. • Write sequences using function notation. • Calculate the sum of n terms in a geometric sequence.	• Short Answer Assignments • Lab • Mastery Exercises • Module Exam
9 – Reflection & Final Exam	• Prepare for exam through discussion board and/or meeting with professor	• Reflection Paper • Final Exam